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# Variable-Flexibility Elastic Model for Digital Image Analysis

**Keywords**: deformable structures, image analysis, object recognition

## ABSTRACT

After a short review of image analysis methods that use deformable structures, this paper reveals that fast and accurate matching can be achieved by employing a variable-flexibility elastic model for image analysis. Methods discussed are illustrated by application examples, with particular emphasis on analysis of wheat grain X-ray images.

## 1. INTRODUCTION

There is always a need for development of robust and efficient tools for high-accuracy quantitative image analysis. The traditional techniques, which employ a series of image processing procedures, e.g. filtering, finding regions of interest, segmentation and quantitative feature extraction – such that the output of one procedure is the input to another [1] – appear to be inefficient and lead to errors in some cases. An example can be image segmentation through thresholding that is accompanied by information loss that consequently causes irreversible errors in optical character recognition. More and more often, one faces problems in which the best image analysis strategy should integrate two or more techniques working virtually in parallel, with continuous mutual exchange of results of their operation. A relatively new generation of image processing tools, based on matching deformable geometric models (deformable structures) to the image contents, is a promising alternative to traditional approaches.

Deformable structures comprise nodes called also active elements. Neighbouring nodes are connected to form flexible curves, grids or discrete deformable surfaces. Image analysis using deformable structures is a process of iterations, called matching process, that systematically displace and deforms the structure to fit an object of interest. All the active elements need initialisation that means, setting preliminary positions at the beginning of the process. Then nodes move over the image being analysed, seeking desired locations. Changing the position of an active element depends simultaneously on image property and locations of neighbouring nodes.

Preliminary work has revealed usefulness of family of deformable structures called active contours or snakes [2] to solve such difficult tasks as boundary detection of heart systole in ultrasound images [3] and quantitative analysis of wheat grains using X-ray images [4]. It was reported in the literature [5], [6] that deformable surfaces can be applied for terrain topography modeling, reconstruction of human-body parts based on tomographic scans, or building CAD models. The technique has proven to be robust in the presence of noise or incomplete data.

The present research effort is focused on elastic models [7] that belong to *model-based recognition* methods. The concept is to compare and match a model from a database with a fragment of image being analysed. The model can be applied for digital image object recognition, for finding object position and aspect, and for object details characterisation.

In this paper, a new two-stage elastic model matching procedure is proposed. A semi-elastic model is used for initial, coarse matching and then highly flexible structure is utilised that allows accurate yet noise-resistant image representation. A new formulation of elastic model equations is introduced, based on hexagonal grid. This simplifies the iterative model formula and leads to savings in computation time.

## 2. ELASTIC MODEL

Elastic model is composed of nodes $\omega$ connected to form a planar graph $\Omega$ (a mesh). Each node carries out partial information on model object features in such a way that entire mesh contains complete colour and shape data.

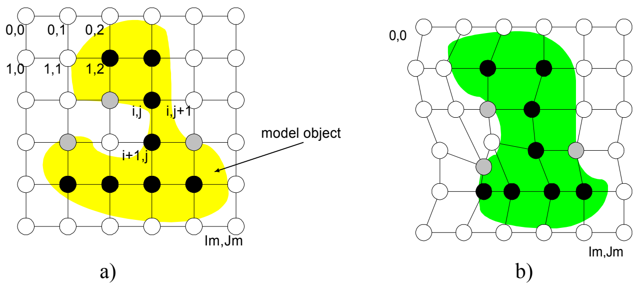



Fig.1. Elastic model: defining model image features at nodes (a), matching process (b).

At the beginning of matching process nodes are placed on a surface of digital image in a form of not deformed graph. Then in successive iterations they move toward object and slightly deform a graph to precisely fit it. Co-ordinates of nodes are $v(\omega,t) = (x(\omega,t), y(\omega,t))$, where $t$ is time (iteration index) time and $x$, $y$ are the corresponding co-ordinate functions. The graph deforms during matching process as a result of minimisation of energy function given by (1). The energy $E_S$ depends on graph shape $E_D$, and image characteristics $E_I$. Analysis of energy value after completion of the matching process allows identification of the object. Lower values of energy indicate larger similarity between the object and its model.

$$E_S = E_D + E_I \tag{1}$$

One of the most important problems, related to the elastic model, is its topology and deformation energy $E_D$ calculation. Energy $E_D$ models internal tensions of elastic graph structure. Therefore, this component limits excessive deformation and is lower when shape of the graph is more regular.

## 3. HEXAGONAL TOPOLOGY

The most common topology is a rectangular graph. Procedures of deformation energy calculations are very complex in such case. For comparison, article [7] describes a method of deformation energy computation using complicated trigonometrical equations and article [8] presents a method of neural network with dynamic link architecture utilised for solving this problem.

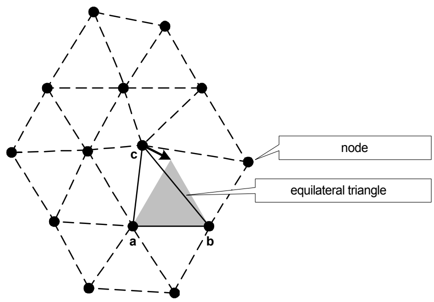
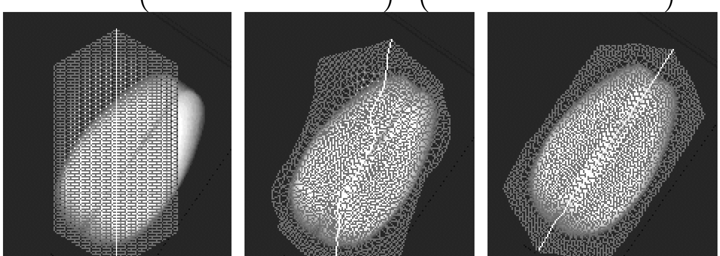



Fig.2. Elastic model based on hexagonal graph.

The contribution of this paper is to use a hexagonal topology (Fig.2) of a graph with originally formulated deformation energy $E_D$. Each node together with neighbouring nodes forms two, three or six triangles. In a not deformed graph, all these triangles are equilateral. Calculation of deformation energy is carried out by integration of partial deformations of triangles in each node. Node deformation coefficient for a single triangle is given by:

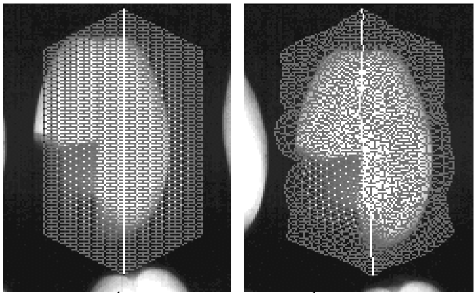
$$\gamma = \left( x_c - \frac{x_a + x_b + \sqrt{3}(y_b + y_a)}{2} \right)^2 + \left( y_c - \frac{y_a + y_b + \sqrt{3}(x_a + x_b)}{2} \right)^2 \tag{2}$$



a) 0th iteration b) 200th iteration c) 360th iteration

Fig.3. Matching process of rotated object

The method of hexagonal-graph elastic model was applied for wheat grain analysis. Result of matching process is presented in Fig.3 and Fig.4.

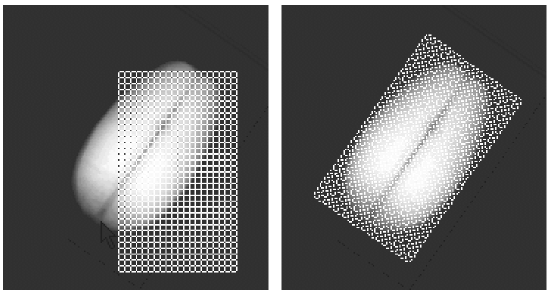


a) 0th iteration b)250th iteration

Fig.4. Matching process of corrupted object

## 4. QUASI-ELASTIC MODEL

Analysis of object using elastic model is quite accurate. The main disadvantage of the method is in large number of iterations needed to achieve the converged result. To circumvent that problem and simplify the matching procedure the quasi-elastic model was developed.



a) 0th iteration b) 40th iteration

Fig.5. Application of quasi-elastic model (rectangular graph)

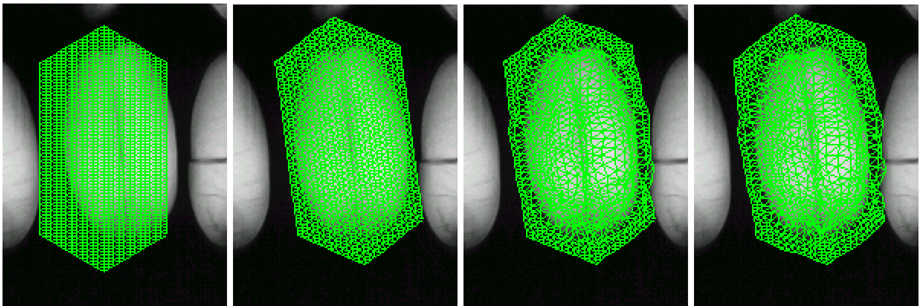
Quasi-elastic model allows only stretching, shifting and rotating the graph as a whole. There are no local displacements in the graph so there is no need to calculate complicated coefficient of deformation energy $E_D$. Therefore, the graph is characterised by three parameters: co-ordinates of its centre $\boldsymbol{D}$, stretching vector $\boldsymbol{R}$ and angle $\alpha$. It can be shown that these parameters are described by following equations:

$$R^t = R^{t-1} + r \sum_{i=0}^{\mathrm{Im}-1} \sum_{j=0}^{Jm-1} \begin{bmatrix} F^h{}_{i,j}\left(i - \frac{I_m - 1}{2}\right) \\ F^v{}_{i,j}\left(j - \frac{J_m - 1}{2}\right) \end{bmatrix} \tag{3}$$

$$D^t = D^{t-1} + d\sum_{i=0}^{\mathrm{Im}-1}\sum_{j=0}^{Jm-1}\begin{bmatrix} F^h{}_{i,j}\cos\left(\alpha^{t-1}\right) - F^v{}_{i,j}\sin\left(\alpha^{t-1}\right) \\ F^h{}_{i,j}\sin\left(\alpha^{t-1}\right) + F^v{}_{i,j}\cos\left(\alpha^{t-1}\right) \end{bmatrix} \tag{4}$$

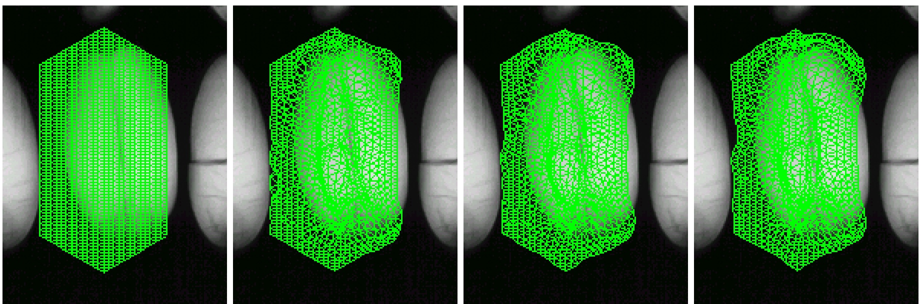
$$\alpha^t = \alpha^{t-1} + a\sum_{i=0}^{\mathrm{Im}-1}\sum_{j=0}^{Jm-1}\left[F^v{}_{i,j}\left(i - \frac{I_m - 1}{2}\right) - F^h{}_{i,j}\left(j - \frac{J_m - 1}{2}\right)\right] \tag{5}$$

where *i, j* are indexes of nodes (see Fig.1), $\boldsymbol{F} = (F^h, F^v)$ is a vector of image influence on the node, *t* is an index of iteration and *d, r, a* are constant values.

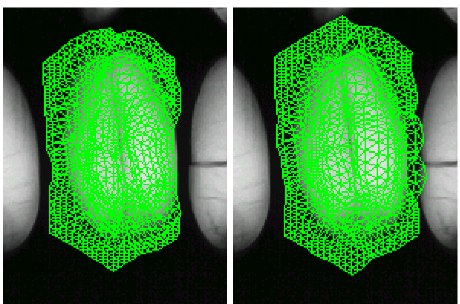


a) 0th iteration b) 10th iteration c) 20th iteration d) 30th iteration

Fig.6. Quasi-elastic model (b) and elastic model (c, d) combined sequentially



a) 0th iteration b) 10th iteration c) 20th iteration d) 30th iteration



e) 80th iteration f) 120th iteration

Fig.7. Application of elastic model exclusively

Result of matching process of quasi-elastic model with rectangular graph is shown in Fig.5. Fig.6 and Fig.7 present comparison of wheat grain image analysis with hexagonal graph applied. Fig.6 presents result of analysis using both quasi-elastic model and elastic model sequentially. Fig.7 shows effect of analysis using elastic model only. It can be noticed that good matching with mixed models is achieved within 20 iterations (ca. 3 seconds with Pentium 120MHz), while elastic model needs approximately 120 iterations (ca. 25 seconds with Pentium 120MHz). It is due to lower number of iterations and computational simplicity of the proposed quasi-elastic model.

## 5. CONCLUSIONS

Deformable-structure methods are efficient and reliable tools of image analysis. Obtained resulting data are easy for further analysis. Presented here in elastic and quasi-elastic models or similar structures are very likely to become a standard tools for object recognition and characterisation. These techniques would be useful in medical diagnostics, robotics, industrial quality control and food industry.

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